Math 53, Discussion 105 and 107

Monday, March 16, 2020 9:06 AM

Topic: 15.1 (iterated integrals) 15.2 (double integrals over rectangles), a little 15.3

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The 3 following problems are from Larsen's (another GSI) worksheet

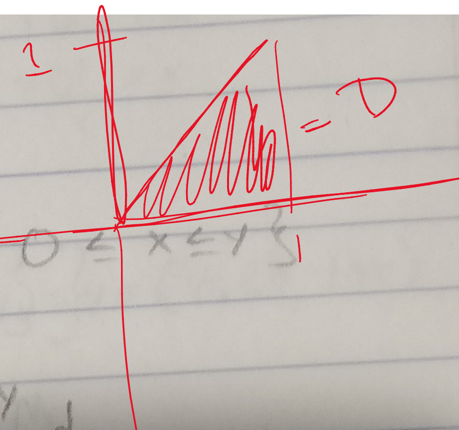
1. Evaluate $\iint_D 2e^{y^2} dA$ for $D = \{0 \le y \le 1, 0 \le x \le y)$, answer should be $e - 1$
2. Compute $\iint_D (x + y) dA$, for $D = \{2 \le x \le 3, 0 \le y \le 4\}$, should be 18
3. Let D be the trapezoid with vertices $(0,0), (2,0), (0,1), (2,3)$, compute $\iint_D x^2 dA$, the answer should be $\frac{20}{3}$

Page 961, 15.2: Fubini's Theorem

$$\int_a^b \int_c^d f(x,y) dy dx = \int_c^d \int_a^b f(x,y) dx\, dy$$

Book problems:

1. 15.2.18: $\int_0^1 \int_0^1 \frac{1+x^2}{1+y^2}$, answer should be $\frac{\pi}{3}$
2. 15.2.35: find the average value of $f = x^2 y$ over the rectangle with vertices $(-1,0), (-1,5), (1,5), (1,0)$



Math Discussion

1) $\iint_D 2e^{(y^2)}\, dA$ for $D = \{0 \le y \le 1, 0 \le x \le y\}$

$$\int_0^1 \int_0^y 2e^{(y^2)}\, dx\, dy = \int_0^1 (2e^{y^2} x)\Big|_0^y\, dy$$

$$= \int_0^1 2ye^{y^2}\, dy = e^{y^2}\Big|_0^1 = \boxed{e-1}$$

2) ... $D = \{2 \le x \le 3, 0 \le y \le 4\}$

2. $\int_2^3 \int_0^4 (x+y)\,dy\,dx$

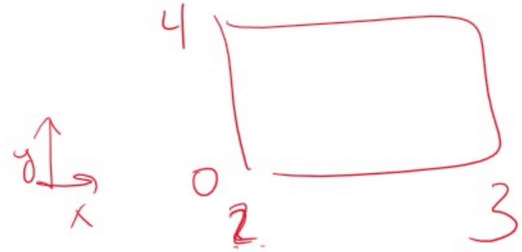
$xy + \frac{y^2}{2} \Big|_0^4$

$\int_2^3 4x + 8 \, dx$

$2x^2 + 8x \Big|_2^3$

$42 - 24$

$\boxed{18}$



Fubini's Theorem:

$\int_a^b \int_c^d f(x,y)dxdy = \int_c^d \int_a^b f(x,y)dydx$ when f is continuous.

MQ Makeup

1) $\int \sqrt{r^2+r'^2}\, d\theta = \int_{\frac{\pi}{2}}^{\pi} \sqrt{9\theta^2+9}\, d\theta = 3\int_{\frac{\pi}{2}}^{\pi} \sqrt{\theta^2+1}\, d\theta$

$\theta = \tan u \quad 3\int$

Math Discussion

1) $\iint_D 2e^{(y^2)}\, dA$ for $D = \{0 \le y \le 1, 0 \le x \le y\}$

$\int_0^1\int_0^y 2e^{(y^2)}\, dx\, dy = \int_0^1 (2e^{y^2}x)\Big|_0^y\, dy$

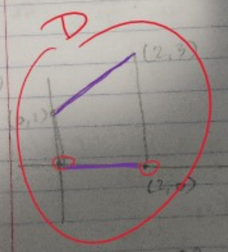
$= \int_0^1 2ye^{y^2}\, dy = e^{y^2}\Big|_0^1 = \boxed{e-1}$

2) $\iint_D (x+y)\, dA$ for $D = \{2 \le x \le 3, 0 \le y \le 4\}$

$\int_2^3\int_0^4 x+y\, dy\, dx = \int_2^3 (xy+\frac{1}{2}y^2)\Big|_0^4\, dx = \int_2^3 (4x+8)\, dx$

$= 2x^2+8x\Big|_2^3 = 18+24-8-16 = \boxed{18}$

3)



$0 \le x \le 2, \; 0 \le y \le x+1$

$\int_0^2\int_0^{x+1} x^2\, dy\, dx$

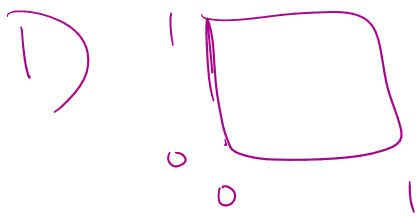
$= \int_0^2 (x^2y)\Big|_0^{x+1}\, dx$

$= \int_0^2 x^2(x+1)\, dx = \int_0^2 x^3+x^2\, dx = \frac{1}{4}x^4+\frac{1}{3}x^3\Big|_0^2$

$= 4 + \frac{8}{3} = \boxed{\frac{20}{3}}$

Book problems:

1. 15.2.18: $\int_0^1 \int_0^1 \frac{1+x^2}{1+y^2}$, answer should be $\frac{\pi}{3}$
2. 15.2.35: find the average value of $f = x^2 y$ over the rectangle with vertices $(-1,0), (-1,5), (1,5), (1,0)$



$$\int_0^1 \int_0^1 \frac{1+x^2}{1+y^2}\,dx\,dy$$

$$= \int_0^1 \frac{x}{1+y^2} + \frac{x^3}{3(1+y^2)} \Big|_{x=0}^{1=x} dy$$

$$= \int_0^1 \frac{1}{1+y^2} + \frac{1}{3(1+y^2)}\,dy$$

$$= \arctan(y) + \frac{1}{3}\arctan(y) \Big|_0^1$$

$$\frac{\pi}{4} + \frac{\pi}{4}\left(\frac{1}{3}\right) = \frac{3\pi + \pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$$

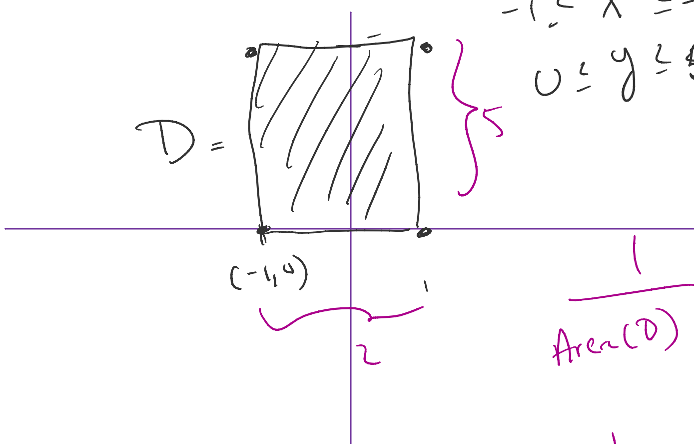
Book problems:

1. 15.2.18: $\int_0^1 \int_0^1 \frac{1+x^2}{1+y^2}$, answer should be $\frac{\pi}{3}$
2. 15.2.35: find the average value of $f = x^2 y$ over the rectangle with vertices $(-1,0), (-1,5), (1,5), (1,0)$

$-1 \le x \le 1$

$0 \le y \le 5$

D =



$$\frac{1}{\text{Area}(D)} \int_0^5 \int_{-1}^1 x^2 y \, dx \, dy$$

$$= \frac{1}{10} \int_0^5 \int_{-1}^1 x^2 y \, dx \, dy$$

$$\frac{x^3}{3} y \Big|_{-1}^{1} = \frac{2}{3} y$$

$$\frac{1}{10} \int_0^5 \frac{2}{3} y \, dy$$

$$\left(\frac{1}{10}\right) \frac{y^2}{3}\Big|_0^5 = \frac{25}{3}\left(\frac{1}{10}\right) = \frac{5}{6}$$